

The Optical Reference Error Analysis and Control Method in Ground Validation System of Stellar-Inertial Integration

Yanqiang Yang, Chunxi Zhang, Jiazhen Lu*, Hao Zhang

The Science and Technology on Inertial Laboratory, School of Instrumentation Science and Opto-electronics Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China.

* Correspondence: ljzbuaa@163.com; Tel.: +86-10-8231-6906-813; Fax: +86-10-8231-6906-818.

Abstract—The authenticity of the Stellar-Inertial integration ground validation system will eventually determine the navigation performance in flight. The optical reference error characteristics remain a crucial but unsolved issue due to the arc-second level accuracy in the ground validation system. The characteristic of the star simulator celestial angle error and the north reference error are discussed and analyzed in detail. A new error control method of the star simulators intersection angle is proposed with the help of the high precision three-axis turntable. The north reference error can be isolated as a certain azimuth bias error in ground validation. The synthesis of all the error sources is performed to give the criterion value of the evaluation accuracy for the ground validation system. Experimental tests confirm that the error analysis and control method is effective. The evaluation accuracy is improved from 10.99 arc-seconds to 4.88 arc-seconds under the condition of remaining the unchanged ground validation system. Meanwhile, such an in ground test system is close to the flight conditions and it can satisfy the stringent requirement for high-accuracy star sensors.

Index Terms—The optical reference, star simulator, ground validation system, the synthesis of error sources.

I. INTRODUCTION

With the development of aeronautics and astronautics, the requirements for attitude measurement are rapidly increasing [1]. The inertial navigation system accuracy is a crucial factor for the completion of a long-time duration missions, whose errors tend to increase over time with unbounded growth [2-4]. Star sensor is an important and promising attitude measurement device with the highest accuracy among different types of attitude measurement devices [5,6]. The Stellar-Inertial integration navigation system can improve the performance of inertial navigation and reduce dependence on other correction methods, all of which are in accordance with the requirements of flight vehicles [7]. Airborne Stellar-Inertial integration navigation system LN-120G by Northrop Grumman Corporation as the most advanced system, its heading accuracy is better than 20 arc-seconds [8]. For the satellite application, the Space Technology Experiment and Climate Exploration satellite mission carries an ASTRO-10 star sensor whose accuracy is close to arc-seconds [9]. Thus, it is essential that the real accuracy of the Stellar-Inertial integration navigation system in flight or in orbit conditions should be verified.

The numerical simulation, hardware in loop simulation and the ground validation are essential to verify the Stellar-Inertial integration method. A numerical simulation tool chain contains the trajectory and sensor emulator [10]. Obviously, the numerical simulation cannot represent the correctness of the new method considering the real error characteristic of the sensors. The hardware in the loop simulation verification method was proposed to resolve the simulation limitations. Quan W [11] adopted the design of software flow integration and hardware functional modularization within actual error characteristic of sensors. German Space

Operations Center also conducted the hardware in the loop tests under simulated flight and laboratory conditions for the SHEFEX-2 mission [12].

The last and important phase is ground test verification and performance evaluation [13]. In terms of inertial navigation system, the accuracy evaluation of position and velocity can be achieved by off-line data processing with differential GPS data or Doppler data [14]. To validate the integrated method of the Stellar-Inertial navigation system, an experiment was conducted by fixing the Strap-down Inertial Navigation System (SINS) and star sensor on a ship in the quiet and stable lake [15]. The attitude accuracy is not directly verified by the position error which is not suitable for the higher attitude accuracy verification.

For arc-seconds level attitude accuracy verification method of the star sensor is generally based on laboratory experiment system and skylight observations, taking the precise motion of the earth as the reference for the static accuracy and the high precision gyro unit in the turntable for the dynamic. The laboratory experiment system is composed of three parts: the star tracker, the three-axis turntable and the star simulator [16]. Ting S [17] proposed a new accuracy measurement method and evaluation criterion for a star sensor based on the direct astronomical which makes the measurement results more authoritative and authentic. Jiang J [18,19] conducted night sky experiments to validate the correctness of the proposed model and method. Zhan D [20] took the laser gyro unit as the gold reference for evaluating the accuracy of the star sensor under highly dynamic conditions in experimental verification. The installation error of the mounting matrix and the error of the gyro unit are the main influence factors of verification accuracy. To solve the dynamic precision evaluation problem owing to dynamic reference value and influence of gyros and accelerometers, Lu [7] proposed a method which is to employ static accuracy as comparison to assess the dynamic performance. In addition, the angular distance constraints and the reference stars in skylight also are selected to serve as golden reference in the calibration procedure [21,22]. The digital orthophoto map and digital elevation model reference data of geometric calibration field also is used for ground processing assessment method for the satellite star sensor/gyro system [23].

So far, there is not enough literature data to focus on the arc-seconds level attitude accuracy verification method of Stellar-Inertial integration system. For the arc-seconds level accuracy evaluation of Stellar-Inertial integration method, the measurement generally is based on auxiliary facilities like the turntable, the theodolite and the optical reference, include star simulator, the north reference. The advantage of the ground validation system compared to the night sky testing can be listed as three key points. Firstly, The turntable can provide the rotation to validation the designed attitude adjustment path for the flight vehicle. In addition, the expensive precision turntable can be taken as a attitude reference to evaluate the stellar-inertial integration method. Finally, the ground validation system can evaluate both the dynamic and static accuracy of the stellar-inertial integration.

However, the relationship between the Stellar-Inertial integration system and auxiliary facilities become complicated due to their

coupling factors, which hinders the ability to achieve optimal verification. The verifiable accuracy of the ground validation system depends on the star simulators mounted at known position with high-precision orientation and the high precision turntable. And it is desirable to analyze errors of the optical reference and accuracy evaluation for the ground validation system of Stellar-Inertial integration method.

In response to the above problems, this paper aims to achieve the following:

(a) Reveal the error characteristic of the star simulators as optical reference;

(b) An improved method is proposed to reduce the star simulators intersection angle error. The synthesis of the total error is implemented to give the accuracy evaluation criterion for the ground validation system.

(c) Based on the error control method, the goal of arc-seconds level accuracy verification under the condition of remaining the primary ground test system for the Stellar-Inertial integration is achieved.

For this research, a new measurement method of the intersection angle between the star simulators was introduced and simulated to verify. In addition, the error budget calculation of the established ground test system is performed by the error synthesizing method. Then the experimental results demonstrate that the proposed method is validated to improve the verifiable accuracy of the Stellar-Inertial integrated method under the condition of remaining the primary ground test system. The error budget calculation is consistent with the repeated experimental results.

II. THE OPTICAL REFERENCE ERROR ANALYSIS

The ground validation system mainly contains the high precision three-axis turntable, the real-time emulator, the Stellar-Inertial navigation system and the optical reference, including the star simulators and the north reference. The turntable is used to simulate the flight vehicle attitude adjustment for the star light measurement. The star simulators as the most critical optical reference are used to simulate the star light in the real sky to correct the attitude error of the navigation system. The north reference provides the north azimuth for the turntable and star simulators. The optical reference error mainly includes the celestial angle error of the star simulator and the azimuth error of north reference.

A. The Stellar-Inertial integration method

In term of the celestial angle integration method, the celestial angles can be obtained in two ways. One is calculated by the product of the star sensor inertial attitude matrix and the SINS position, the other is obtained directly from SINS attitude. The differences of the celestial angles ΔEl and ΔAz are the corresponding measurement information of this integration method, which can be written as [24],

$$\Delta El = El_{n(s)} - El_{n(c)}, \Delta Az = Az_{n(s)} - Az_{n(c)} \quad (1)$$

where El_{ref} and Az_{ref} are called the celestial angles in the reference frame, including the computer-frame $n(c)$ which is the local level frame at the computed position and the platform frame $n(s)$ which is the navigation frame built by the computed attitude of the SINS.

The observation equations showed as follows in reference [21],

$$\mathbf{Z} = \begin{bmatrix} \Delta El \\ \Delta Az \end{bmatrix} = \begin{bmatrix} -\cos Az_{n(c)} & \sin Az_{n(c)} & 0 \\ -\tan El_{n(c)} \sin Az_{n(c)} & -\tan El_{n(c)} \cos Az_{n(c)} & 1 \end{bmatrix} \cdot \begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} \quad (2)$$

Where $\mathbf{Z}$ is the measurement vector of the celestial angle integration method. The platform angle error

vector $\begin{bmatrix} \psi_{n(s)x} & \psi_{n(s)y} & \psi_{n(s)z} \end{bmatrix}^T$ can be uniquely solved by two star vectors observation. In addition, the platform error vector should be divided into the initial platform error angle vector $\begin{bmatrix} \psi_{0x} & \psi_{0y} & \psi_{0z} \end{bmatrix}^T$ and the time-varying platform error angle vector $\begin{bmatrix} \psi_{n(s)x}(t) & \psi_{n(s)y}(t) & \psi_{n(s)z}(t) \end{bmatrix}^T$ by the gyro errors in movement as follow.

$$\begin{bmatrix} \psi_{n(s)x} \\ \psi_{n(s)y} \\ \psi_{n(s)z} \end{bmatrix} = \begin{bmatrix} \psi_{n(s)x}(t) \\ \psi_{n(s)y}(t) \\ \psi_{n(s)z}(t) \end{bmatrix} + \begin{bmatrix} \psi_{0x} \\ \psi_{0y} \\ \psi_{0z} \end{bmatrix} \quad (3)$$

B. The principle of the ground validation system

The Stellar-Inertial navigation system is installed on the three-axis high precision turntable. The turntable rotation can simulate the navigation system attitude adjustment to conduct the star light observation. Meanwhile, more than two star simulators are mounted on the stable base to simulate the real sky star light. It can be assumed that the turntable and the star simulators have a high accuracy close to the arc-seconds level. The emulator conducts real time data acquisition and navigation solution. Finally, the attitude error is calculated by the precision turntable reference, and the repeated tests also should be conduct to provide sufficient verification. The celestial angles of the star simulators should be measured by the theodolite whose error can be introduced in the ground validation system. Hence, it is hard to establish the arc-seconds level optical reference.

C. The optical reference error analysis

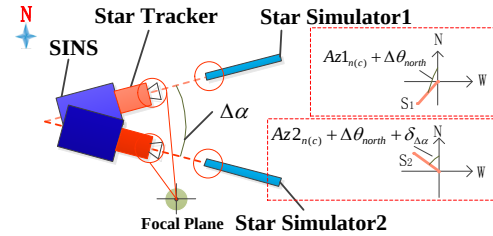


Figure 1 The diagrammatic sketch of the intersection angle

In the ground validation system, the azimuth error of the optical reference, star simulators, mainly expressed as two parts. One is the certain error of the north reference which is called as $\Delta \theta_{north}$. In the ground validation system, the first star simulator azimuth is measured based on the north reference. The azimuth error and the north reference error are not independent each other and hard to decouple. After the theodolite measurement, the azimuth error of the first star simulator is a kind of certain error in accordance with the characteristic of the north reference error. So they are collectively called as $\Delta \theta_{north}$. Another is the intersection angle error between the star simulators as $\delta_{\Delta \alpha}$. So the measurement value of the azimuth angle expressed as $Az1_{n(c)}$ and $Az2_{n(c)}$ of the star simulator by the theodolite can be written as.

$$\begin{cases} Az1_{n(c)} = Az1_{n(c)} + \Delta \theta_{north} \\ Az2_{n(c)} = Az1_{n(c)} + \Delta \alpha + \Delta \theta_{north} + \delta_{\Delta \alpha} = Az2_{n(c)} + \Delta \theta_{north} + \delta_{\Delta \alpha} \end{cases} \quad (4)$$

Where $\Delta \alpha$ is the truth value of the intersection angle between the two star simulators as described in the Figure 1. The $Az1_{n(c)}$ and $Az2_{n(c)}$ is the truth azimuth value of the star simulate without error.

The elevation error $\Delta_{El1(c)}$ and $\Delta_{El2(c)}$ of the two star simulators can be reduced to the minimum value by adjusting to local horizontal plane with the aid of electronic theodolite. The measurement elevation value of the star simulator should be zero arc-seconds. The elevation precision of the theodolite is called as δ_{lev} which is less than 1 arc-second. The limit error of the elevation angle of the star simulator in geographic coordinate frame can be expressed as,

$$El1_{(c)} = \Delta_{El1(c)} < \delta_{lev}, El2_{(c)} = \Delta_{El2(c)} < \delta_{lev} \quad (5)$$

In the ground validation system for the Stellar-Inertial integration, the azimuth error and the intersection angle error between the star simulators is the key factor to affect the verification accuracy. Generally, the ground test procedure contains initial alignment, posture adjustment and measuring the star vector several times. The star simulator is adjusted to the horizontal plane and the elevation angle is close to zero. In the first measurement time of the Stellar-Inertial system, the observation equation Eqs.(2) can be simplified as below,

$$\Delta Az_1 + \Delta \theta_{north} = \psi_{0z} \quad (6)$$

Obviously, the observation value owing to the azimuth error of the first star simulator observation is equal to the initial alignment error. In the Eqs.(4), the azimuth error of the first observation star simulator is defined as the certain error of the north reference. It can be projected in the equivalent eastward gyroscope constant bias and given a wrong gyroscope bias estimation value if the north reference certain error is larger than the alignment accuracy. But it can be omitted when the north reference certain error is about several arc-seconds.

For the second star simulator measurement, the observation equation can be given as,

$$\Delta Az_2 + \Delta \theta_{north} + \delta_{\Delta\alpha} = \psi_{0z} + \psi_{n(s)z}(t) \quad (7)$$

Substitute the Eqs.(6) into the Eq.(7), the initial alignment azimuth error can be cancellation, as Eqs(8),

$$\Delta Az_2 - \Delta Az_1 + \delta_{\Delta\alpha} = \psi_{n(s)z}(t) \quad (8)$$

Here, the intersection angle error between the star simulators is directly related as the time-varying platform error angle introduced by the gyro errors. For the Stellar-Inertial integration method, the random constant errors of the gyroscope would be erroneously estimated owing to the intersection angle error. Thus, the accuracy deterioration of the integration method will appear in tests.

In the above section, the star simulators error characteristic have been analyzed in two parts and the numerical simulation verification should be addressed in the below. In order to improve the accuracy of the ground validation system, we assume that the Stellar-Inertial navigation system cannot perform pose adjustment when the star sensor conducts measurements. In addition, at least seven times measurements are needed, as there are fourteen unknown parameters for the inertial device error model and two equations for each measurement. A calibration route with seven times star light observations for the gyros error and the star sensor installation errors are described as Figure 2.

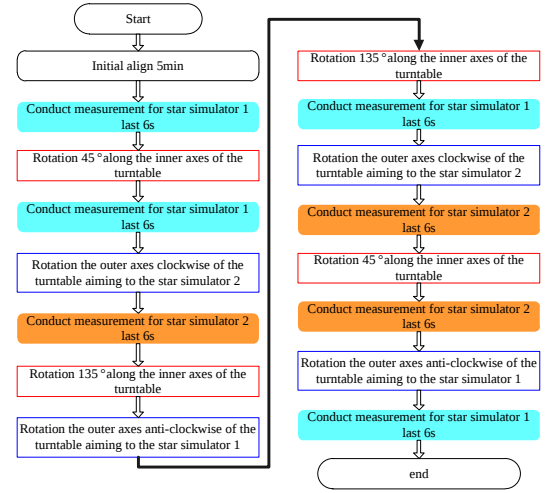


Figure 2 The flowchart of the seven times measurements

The component of Stellar-Inertial navigation system is the combination consisting of high-precision laser strap inertial measure unit and star sensor, which is aim to realize high precision automatic navigation. Numerical simulation results for the ground validation system based on the seven times star observation is presented. Covariance analysis is employed in this section.

The covariance analysis conditions are set as follows. The constant drift of the gyro is 0.03 °/h. The scale factor error of the gyro is 2×10^{-5} . The measurement noise of the gyro is Gaussian white noise with an amplitude of $0.006^\circ/\sqrt{h}$. The misalignment errors of the gyro are approximately 10 arc-seconds. The accelerometer bias is 5×10^{-5} g. The scale factor error of the accelerometer is 5×10^{-5} . The accelerometer measurement noise is Gaussian white noise with an amplitude of 1.5×10^{-5} g. The misalignment errors of the accelerometer are approximately 5 arc-seconds. The gyro and accelerometer measurements are generated for a sample rate of 100 Hz.

The star sensor measurement noise is Gaussian white noise with an amplitude of 5 arc-seconds. The field of star sensor view is about 2° . The star sensor measurement update is provided at a frequency of 10 Hz. The displacement angle of star sensor is about 30 arc-seconds.

In the above, the azimuth error of the golden reference, star simulators, mainly expressed as two parts. Four simulation cases are conducted to reveal the error characteristic. The elevation angle error is omitted in this section in order to isolate the azimuth error influence. The four cases can be described as below,

Case-1: Simulation without the star simulator error;

Case-2: Simulation considering the first part error which called as the north reference error about 5 arc-seconds.

Case-3: Simulation considering the second part error which called as the intersection angle error about 10 arc-seconds.

Case-4: Simulation considering the two kinds of errors. The north reference error is about 5 arc-seconds and the intersection angle error is about 10 arc-seconds;

The setting value of the optical reference error for the four simulation cases refer to the actual ground system. For example, the north reference error is 7 arc-seconds. The date is 30 months from last calibration in our own lab. The intersection angle error between the star simulators is set as 10 arc-seconds.

In this simulation, we consider the gyro all-parameter error model consisting of three terms which include non-orthogonality, scale factors and biases. In addition, assuming a small error perturbation in the SINS dynamic system, a KF is used for estimation and correction of the system errors. The state vector includes all the gyro errors and the star sensor installation errors. The KF's measurement update interval was 0.1 s and the prediction interval was 0.01 s. To keep the

paper reasonably concise, equations of Kalman Filtering are presented in reference [25] and the all-parameter error model is presented in reference [26].

In order to verify the integration method and the flowchart of the star light measurement in the ground test, covariance analysis is conducted. The standard deviation (STD) convergence value of the error states were shown in Table 1.

Taking as an example, the STD of all errors is represented as the degree of observability and listed in the Table 1. All of the errors can be estimated and the platform error should be less than 1.64 arc-seconds in the case of the flowchart of the star light measurement. In addition, the four cases of numerical simulation have revealed the characteristic of the star simulator error in accordance with the previous analysis. For the case-1, the platform error is less than the STD value due to without considering the star simulator error. In the case-2, considering the north reference error of the first star simulator, a certain azimuth error is added in the platform error equal to the north reference error, and other platform errors angles and the device errors are not influenced in the integration method. On the contrary, the intersection angle error between the two star simulators is irrelevant with the platform errors, but related with the gyroscope scale estimating value. Obviously, the case-4 can be seen as the composition of the case-2 and the case-3.

According to the numerical simulation, the star simulator optical reference error is classified into two parts which can be isolated based on the different characteristic of the error. And the intersection angle error is unrelated to the north reference error of the star simulator in the integration method.

Table 1 The standard deviation (STD) convergence value and estimated value of the error states in four cases

Error states	STD value	Case-1	Case-2	Case-3	Case-4
Attitude error					
$\psi_x / (^\circ)$	1.2784	-0.65	-0.65	-0.66	-0.68
$\psi_y / (^\circ)$	1.6359	-0.03	-0.04	-0.03	-0.02
$\psi_z / (^\circ)$	0.7108	0.76	5.77	0.80	5.81
Estimated Value					
$gBx / (^\circ/h)$	0.0049	0.0302	0.0305	0.0306	0.0307
$gBy / (^\circ/h)$	0.0049	0.0284	0.0287	0.0289	0.0279
$gBz / (^\circ/h)$	0.0279	0.0054	0.0054	0.0072	0.0071
$gSFx / (ppm)$	2.19	18.35	18.32	-12.27	-12.29
$gSFy / (ppm)$	3.29	19.20	19.20	19.13	19.28
$gSFz / (ppm)$	1.66	20.69	20.69	-10.01	-10.02
$gMAxz / (^\circ)$	0.50	10.00	10.01	10.01	10.02
$gMAyx / (^\circ)$	1.05	9.87	9.87	9.84	9.84
$gMAzy / (^\circ)$	1.05	9.64	9.64	9.62	9.64
$\mu_x^b / (^\circ)$	1.36	29.23	29.23	29.25	29.26
$\mu_z^b / (^\circ)$	1.19	30.93	30.93	30.92	30.92

III. THE ERROR CONTROL METHOD AND THE ERROR SYNTHESIS OF THE GROUND VALIDATION SYSTEM

The objective of ground validation system is to evaluate accuracy and achieve the arc-seconds level accuracy verification. The star simulators as optical reference are the most critical components in the ground validation system. This section put forward a new measurement method of the optical references. The synthesis of the total system error is performed to give the limit error of the ground validation system. According to this error control method, the arc-seconds level accuracy is achieved under the condition of the unchanged system hardware.

A. The north reference and the high precision three-axis turntable

Generally, the north reference is necessary in the process of the navigation system ground test. It can be used to measure the azimuth angle of the star simulators by the theodolite and correct the azimuth angle of the high precision turntable. Generally, the north reference error is less than 1 arc-second. The angular position repeatability of the turntable is better than 1 arc-second.

B. Construct the star simulators

The star simulator as optical reference should be mounted at known position. The height of the star sensor and star simulator should be equal to each other as Eq(9). It can ensure that the star light can be observable by star sensor.

$$h_{st} = h_{ss} \quad (9)$$

Obviously, the star simulator should be adjusted to the horizontal plane for the minimum error of the inclination angle. If the star light of the star simulator is in horizontal plane, the inclination angle of star sensor should be less than the half of the field view. If the center view of the star sensor is in horizontal plane, the star sensor inclination should meet the follow inequation,

$$D/2 \geq l \cdot \tan(\theta_{ss}) \quad (10)$$

Where D is the lens bore diameter of the star sensor. l is the distance between star sensor and simulator. θ_{ss} is the inclination angle of star simulator.

Aiming to verify the Stellar-Inertial integration method, more than two star simulators as gold reference can solve the platform error angles. The star light vector in geographic coordinate frame can be characterized by the celestial azimuth angle $Az_{n(c)}$ and the celestial elevation angle $El_{n(c)}$ in geographic coordinate frame. The elevation angle cannot be measured accurately unless adjusted to local horizontal plane with the aid of electronic theodolite.

The limit error of the elevation angle of star simulator in geographic coordinate frame can be expresses as,

$$\Delta_{El1n(c)} = \Delta_{El2n(c)} < \delta_{lev} = 1'' \quad (11)$$

The azimuth angle also should be measured by the theodolite and the north reference or the gyro theodolite. Considering the cost of the ground test, the type of DJ2 theodolite is used to optical aiming, which accuracy less than 2 arc-seconds in single measurement.

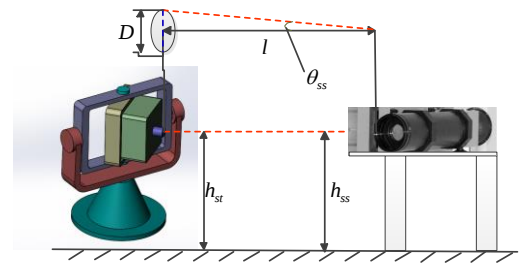


Figure 3 The installation sketch map of the star simulator

Apparently, the accuracy of the intersection angle between star simulators as optical reference is the key factor in the ground test. There are two kinds of methods to measure the azimuth angle. One is to measure the azimuth angle separately. The other is to measure the intersection angle and one of the azimuth angles. The error of theodolite is unavoidable during the azimuth angle measurement procedure of each star simulator. The azimuth angle of each star simulator need be measured by two electronic theodolites in four times measurement due to space limitation.

The total limit error of the each azimuth angle of the star simulator can be list as,

$$\Delta_{Az1n(c)} = \Delta_{Az2n(c)} = \pm \sqrt{\Delta\theta_{north}^2 + 4*\delta_{th}^2} \quad (12)$$

Where the north reference error $\Delta\theta_{north}$ is the certain system error, and the δ_{th} is the random error of the theodolite in single measurement.

Although the azimuth error of each star simulator is large, it can be classified in two parts base the above section II. The azimuth error of the first star simulator can be isolated in the repeatability tests because of the certain characteristic. The repeatability of many tests should not be affected by the north reference error.

The intersection angle between the two star simulators also can be measured by two electronic theodolites in four times measurement and the total limit error of the intersection angle can be written as

$$\delta_{\Delta\alpha} = \pm \sqrt{4*\delta_{th}^2} \quad (13)$$

In term of the J2 type thodolite, and the north reference error is about 1 arc-second. The limit error of the star simulator is larger than the star sensor accuracy of the star sensor, it can be calculated as ,

$$\Delta_{Az1n(c)} = \Delta_{Az2n(c)} = \pm 4.13'', \delta_{\Delta\alpha} = \pm 4'' \quad (14)$$

In order to further improve the accuracy of the star simulator azimuth angle and the intersection angle, a new method by the high precision three-axis turntable and the star sensor is proposed. Reset the turntable, and then rotate the outer axis of the turntable, the optical axis of star sensor points to the star light. Adjusting the pitch angle of the turntable, it should be well fine until the star point in the focal plane nearest to the original point for high accuracy measurement of the star sensor. The azimuth of the turntable can be stored as θ_{tun1} and it can be developed as Eqs.(15). Similarly, the

azimuth of the turntable can be stored as θ_{tun2} and it is expressed as Eqs (16), when the star sensor optical axis points to another star simulator. The measurement error of star sensor can be omitted owing to the star vector close to the center of the CCD plane.

$$\theta_{tun1} = \theta_{tun1} + \delta_{\theta_{pos}}, \theta_{tun2} = \theta_{tun2} + \delta_{\theta_{pos}} \quad (15)$$

The elevation angle of star simulator in geographic coordinate frame can be measured by the theodolite. The truth value of the intersection angle between the star simulators can be obtained as

$$\Delta\alpha = \text{abs}(\theta_{tun1} - \theta_{tun2}) \quad (16)$$

Assume that each error of the azimuth of the turntable obeys the normal distribution. The total synthetic error of the intersection angle between star simulators can be expressed as below.

$$\delta_{\Delta\alpha} = \sqrt{\delta_{\theta_{pos}}^2 + \delta_{\theta_{pos}}^2} = \pm \sqrt{2} * \delta_{\theta_{pos}} \quad (17)$$

Where $\delta_{\theta_{pos}}$ is the position error of the turntable and it is uncertainly and close to 1 arc-second. Obviously, the limit error of the intersection angle has been largely reduced to $\pm \sqrt{2}''$ by proposed method.

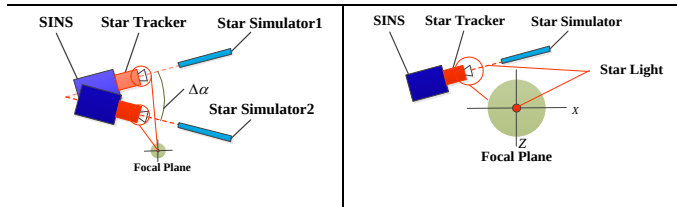


Figure 4 The star sensor view in the process of the intersection angle measurement

C. The error of the ground validation system environment

The error caused by the external environment includes vibrations of the three-axis turntable and the star simulators. In addition, the

performance of the Stellar-Inertial navigation system is affected by test temperature. According to the specification of the ground validation system, the air conditioner should be used to keep the test system at a constant temperature and it would be better finish the ground test at night.

D. The synthesis of the total system error

Error calculations are a specialized form for accuracy evaluation of ground test system of Stellar-Inertial integration. They determine the separate effects of individual error sources, or groups of error sources, which are thought to have potential influence on ground test system accuracy. In the term of the two star simulators, the total limit error in the two times measurement information can be expressed as,

$$\Delta_Z = \pm \sqrt{\Delta_{\Delta E1}^2 + \Delta_{\Delta A1}^2 + \Delta_{\Delta E2}^2 + \Delta_{\Delta A2}^2} \quad (18)$$

Where $\Delta_{\Delta E1}$, $\Delta_{\Delta A1}$, $\Delta_{\Delta E2}$, $\Delta_{\Delta A2}$ is the limit error of the measurement information and each error is independent of each other, the Δ_Z is the total limit error. Considering the corresponding measurement equation (2), the single limit error can be list as follow,

$$\Delta_{\Delta E1} = \pm \sqrt{\Delta_{El1n(c)}^2 + \delta_{El1n(s)}^2}, \Delta_{\Delta E2} = \pm \sqrt{\Delta_{El2n(c)}^2 + \delta_{El2n(s)}^2} \quad (19)$$

$$\Delta_{\Delta A1} = \pm \sqrt{\Delta_{Az1n(c)}^2 + \delta_{Az1n(s)}^2}, \Delta_{\Delta A2} = \pm \sqrt{\Delta_{Az2n(c)}^2 + \delta_{Az2n(s)}^2}$$

Where $\delta_{El1n(s)}$ and $\delta_{Az1n(s)}$ are the uncertainly error owing to the residual errors of SINS and the installation errors of star sensor. The north reference error is both coupled in the measurement information $\Delta_{Az1n(c)}$ and $\Delta_{Az2n(c)}$. The uncertainly error $\delta_{El1n(s)}$ and $\delta_{Az1n(s)}$ also are both coupled in the measurement information $\Delta_{Az1n(s)}$ and $\Delta_{El1n(s)}$.

But, the error relativity among the measurement information can be eliminated by the linear transformation. So one of the errors should be considered once in the synthesis process. The total error of the measurement information can be considered as below.

$$\Delta_Z = \pm \sqrt{\Delta_{\Delta E1}^2 + \Delta_{\Delta A1}^2 + \Delta_{\Delta E2}^2 + \Delta_{\Delta A2}^2} = \pm \sqrt{F^2(\delta_{\epsilon_g}, \delta_{\mu_g}) + \Delta_{El1n(c)}^2 + \Delta_{El2n(c)}^2 + (\Delta\theta_{north}^2 + 4*\delta_{th}^2) + \delta_{\Delta\alpha}^2} \quad (20)$$

For the improved method, the intersection angle and the azimuth of the first star simulator is precisely measured. The attitude adjustment path proposed in Table 2 is used to estimate the device errors. Note that the gyro estimation error is covered by covariance and the estimation precision is up to 90% in Table 1 in the case of a specific path.

$$\sqrt{\delta_{El1n(s)}^2 + \delta_{Az1n(s)}^2} < F(\delta_{\epsilon_g}, \delta_{\mu_g}) \approx 30'' * 10\% = 3'' \quad (21)$$

Substitute the equation (11), (12), (17), (21) in the equation (20), the total limit error of the observation information based on the improved method should be calculated as,

$$\Delta_Z = \pm 5.47'' \quad (22)$$

The least squares estimation of the state can be written as,

$$\mathbf{Z} = \mathbf{H} \cdot \mathbf{X} \Rightarrow \mathbf{X} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Z} \quad (23)$$

Based on the linear projection theorem, the inequality of the variable state limit error can be express as

$$\|\delta_x\| \leq \|(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T\| \cdot \|\delta_z\| \quad (24)$$

We can definite the amplitude parameter of error as below

$$k = \|(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T\| \quad (25)$$

To analyze the effects of the intersection angle between the two star simulators, we take the following numerical examples when,

$$Az_{n(c)1} = Az_{n(c)2} + \Delta\alpha, (\Delta\alpha = [0^\circ, 360^\circ]) \quad El_{n(c)1} = El_{n(c)2} = 0^\circ \quad (26)$$

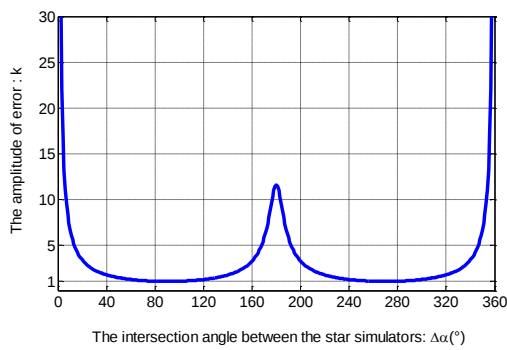


Figure 5 The relationship between the amplitude of error and the intersection angle

It can be seen from the Figure 5 that the optimal intersection angle should be 90 degree. The two star simulators are orthogonal each other. The state variable limit error is close to the measurement limit error then the amplitude of error is near to 1. And in the case of the method mentioned above, the platform angle error should be less than 5.47 arc-seconds.

IV. EXPERIMENTAL TESTS

To validate the feasibility of the improved method, experiments were performed based on the ground validation system as shown in Figure 6. The Stellar-Inertial integrated navigation system is strap down installation by fixing the laser strap inertial measure unit and star sensor. The installation error of star sensor and other device errors will be on-line calibrated in the test procedure. The body frame of the laser strap inertial measure unit which has been calibrated and reclosed with the turntable frame. Placing and fixing the star simulators, the intersection angle should be about 90°. Install the Stellar-Inertial navigation system on the three-axis turntable, and utilize the precise location function of the turntable to adjust the navigation system to horizontal plane. The prepare tests is needed to validate the simulator star light can be projection in the center of the star sensor image plane to measure the intersection angle between the star simulators. The LINKS-BOX can be worked as a real-time emulator to collect data and control three-axis turn table. The characteristic of experimental instruments in ground test system of Stellar-Inertial integration is listed in the Table 2.

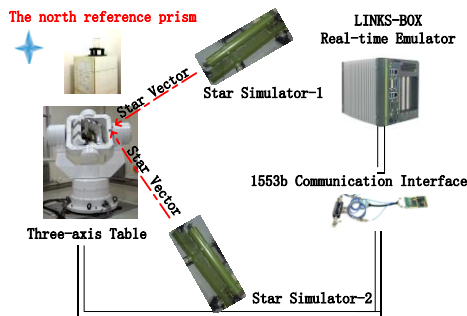


Figure 6 Schematic diagram of the ground validation system
Table 2 The characteristic of experimental instruments in ground test system of Stellar-Inertial integration

Experimental instruments	Description(3 σ)		
The high precision three axis turntable	Angular position accuracy	1 arc-second	
The laser strap inertial measure unit		Gyro	Accelerometer
	Bias drift	0.03 °/h	50ug
	Random walk	0.006°/√h	-

	Scales	20ppm	50ppm
	The installation error	5 arc-seconds	
	Sampling frequency	100Hz	
The single star sensor	The field of view	3 °	
	Sampling frequency	10 Hz	
	The pointing accuracy	5 arc-seconds	
	The installation error	200 arc-seconds	
The star simulator	The collimation	1 arc-second	
The type DJ2 optical theodolite	Horizontal accuracy	±1 arc-seconds	
	Azimuth accuracy	±2 arc-seconds	
The north reference	1 arc-second (After measurement)		

A. Experimental tests

The improved measurement method of the optical reference, takes the high precision position of the turntable and the high precision north orientation of the turntable as the reference for attitude measurements. In use, the Stellar-Inertial integrated navigation system is fixed on the turntable and moves along with the specified path detailed in Figure 2. In the process of specific path movement, the star sensor conducts to measure the star simulator seven times to correct the attitude. After movement, the turntable stops to maintain the fixed position. The used view angle of the star sensor is less than 1.5°. Ten repeatability tests have been finished and three cases have been conducted to verification the improve method. The three cases is defined as,

Case1: Establish the golden reference using the conventional method;

Case2: The intersection angle between the two star simulators is measured by the improved method;

Case3: The north reference azimuth is instead of the new value measured by the professional method before the validation tests;

The Stellar-Inertial integration navigation system conducts ground initial alignment and poses adjustment and measures the star vector seven times according to the movement path. The repeated experimental tests were finished. The attitudes at the end of the test are illustrated in Table 3. The last time position of the turntable remains to the angle $[0.476^\circ \ 0.013^\circ \ 313.954^\circ]$ which is point to the first star simulator. The three axis turntable attitude can be as reference to evaluate the accuracy of the ground validation system. The multiple repeatability experiments are used to verify the Stellar-Inertial integration navigation accuracy and the integration method. The average attitude error is defined as the error of the average attitude value compared to the turntable attitude. The Max-Min error is defined as the error range of each attitude angle. The repeatability is defined as the norm of the Max-Min error vector. The max total error is calculated as the max attitude error of the multiple tests.

For the case 1, the error of the average attitude in the ten repeatability tests is about $[-0.81'' \ 1.14'' \ 8.42'']$. It is much larger than the star sensor accuracy and not useful to verify the accuracy of the Stellar-Inertial integration. The repeatability of the multiple tests is expressed as the Max-Min value about 7.49''. The max total error is about 10.99'' which is larger than the analysis total limit error 5.47''.

For the case 2, the intersection angle between the star simulators was measured by the improved method using the three-axis turntable and star sensor. The attitude error of the average attitude in the ten repeat tests is about $[-0.86'' \ 1.04'' \ 8.35'']$. Although the attitude error is not improved, the repeatability of the multiple tests is reduced from 7.49'' to 5.52''. The max total error is reduced from 10.99'' to 9.83''. The certainly azimuth error owing to the north reference is the main error source in the attitude error.

In order to further improve the precision of the average attitude and the repeatability of the multiple tests, the north reference of the prism is calibrated before the tests on a clear night. The attitude repeatability is approximately 5.4" in the case 3, verifying the correctness of the error analysis and the total synthetic error, in good agreement with the analyze result. The max total attitude error 4.88" is less than the analysis result 5.47".

Compared to the case-3, the certainly azimuth orientation error of the case-2 is mainly come from the optical north reference. For the case-2, the average heading attitude error in the multiple repeat tests contains the certainly azimuth orientation error introduced by the optical north reference. Obviously, the optical north reference error is not affect the attitude repeatability. For example, the repeatability of the multiple tests in case-2 is slightly in agreement case-3. But the total error is directly influenced by the intersection angle error between the star simulators. In the case-3, the attitude repeatability is

close to the limit error of the theory analysis of the ground system. The max total attitude error is reduced from 10.99 arc-seconds to 4.88 arc-seconds under the condition of remaining the primary ground test system.

In order to verify the correctness of the Stellar-Inertial integration method, the estimation value of all the states also are listed in Table 4. It can conclude that the repeatability of the device errors in multiple tests are in agreement with the characteristic of the Stellar-Inertial navigation system in the case of specific path movement made the error observable. Comparison with the simulation STD value of the states in the Table 1, the gyro bias of the z axis and the gyro scale error of the y axis have poor observability in the terms of other errors. Thus, the Max-Min value of the gyro bias of the z axis and the gyro scale error of the y axis also are larger than other errors.

Table 3 The repeatability tests of accuracy verification in ground test system of Stellar-Inertial integration

Tests Num	Case-1			Case-2			Case-3		
	Pitch/°	Roll/°	Head/°	Pitch/°	Roll/°	Head/°	Pitch/°	Roll/°	Head/°
1	0.47583	0.01232	313.95665	0.47597	0.01395	313.95585	0.47596	0.01397	313.95441
2	0.47569	0.01279	313.95624	0.47576	0.01336	313.95644	0.47576	0.01337	313.95491
3	0.47549	0.01314	313.95587	0.47546	0.01364	313.95567	0.47546	0.01367	313.95431
4	0.47561	0.01356	313.95668	0.47587	0.01315	313.95638	0.47586	0.01317	313.95491
5	0.47558	0.01322	313.95617	0.47547	0.01326	313.9564	0.47546	0.01327	313.95491
6	0.47549	0.01358	313.95561	0.47586	0.01315	313.95659	0.47586	0.01317	313.95521
7	0.47604	0.01358	313.9562	0.47577	0.01314	313.95628	0.47576	0.01317	313.95491
8	0.47586	0.01369	313.95679	0.47586	0.01295	313.95629	0.47586	0.01303	313.95491
9	0.47623	0.01358	313.95699	0.47576	0.01293	313.95659	0.47576	0.01307	313.95521
9	0.47603	0.01358	313.95621	0.47576	0.01293	313.95659	0.47578	0.01327	313.95531
10	0.47566	0.01345	313.95665	0.47576	0.01326	313.95671	0.47576	0.01325	313.95529
Mean-value	0.47578	0.013317	313.9563	0.47576	0.01329	313.95632	0.47575	0.01331	313.9549
Average attitude error/"	-0.81	1.14	8.42	-0.86	1.04	8.35	-0.89	1.12	3.36
Max-Min error/"	2.66	4.93	4.97	1.83	3.63	3.73	1.8	3.6	3.6
The repeatability/"	7.49			5.52			5.4		
The max total error/"	10.99			9.83			4.88		

Table 4 The estimation value of all attitude related device errors in the Stellar-Inertial integration

Tests Num	Gyro bias/(°h)			Gyro scales/(ppm)			The installation error/(°)			/(°)	
	gBx	gBy	gBz	gSFx	gSFy	gSFz	gMAxz	gMAyx	gMAzy	μ_x^b	μ_z^b
1	0.0074	-0.0003	0.0078	6.6342	-0.9144	2.7743	3.4498	4.2334	2.2554	-169.3261	101.0328
2	0.0082	-0.0008	0.0067	8.3481	-7.6989	-3.1961	3.9759	7.3178	3.9025	-168.2360	102.4891
3	0.0030	-0.0017	0.0080	7.4935	-3.2373	1.1305	4.0746	5.4936	2.1265	-169.7234	101.3777
4	0.0057	-0.0040	0.0006	6.3003	-4.8278	0.9324	5.7469	6.2568	4.0144	-168.5022	100.4580
5	0.0024	-0.0029	0.0028	6.5307	-3.3741	1.2898	5.8510	5.6933	4.2227	-168.5851	100.8452
6	0.0093	-0.0007	-0.0067	6.8770	-0.5054	0.3910	6.4633	5.3004	3.9966	-168.6469	100.1229
7	0.0104	0.0020	-0.0031	6.9919	-1.4214	1.4344	6.3455	5.9418	3.6823	-169.4075	100.7509
8	0.0084	-0.0011	-0.0032	6.5286	-4.3227	2.5811	6.3281	6.2307	3.8870	-169.5162	100.1369
9	0.0050	0.0008	-0.0076	6.9561	-3.0606	0.0103	7.1459	6.1294	5.4442	-168.3289	100.4854
10	0.0056	-0.0026	-0.0028	6.9608	-0.9228	-0.4175	7.0441	5.2515	3.8930	-168.4832	100.6169
Max-Min value	0.0080	0.0060	0.0156	2.0478	7.1935	5.9704	3.6961	3.0844	3.3177	1.4874	2.3662

B. Discussion

In this section, experimental tests are performed to validate the feasibility and effectiveness of the proposed improved method of the ground validation system for Stellar-Inertial integration. The synthesis of the total system error is conducted to give the limit error of the ground validation system under the condition of remaining the primary ground test system. The experimental results prove the Stellar-Inertial integration attitude accuracy coincides with numerical simulation analysis conclusion. The major contributions of this study are listed below.

(a) The azimuth error of the two star simulators is described as the equivalent north reference error and intersection angle error. It is easy to analysis the error characteristic in the ground validation system of the arc-seconds level;

(b) All errors are considered in the analysis of the limit error of the ground system. And the evaluation criterion for the ground validation system is proposed in the study.

(c) Achieve the arc-seconds level accuracy verification for the Stellar-Inertial integration method under the condition of the remaining the primary ground test system.

In conclusion, the improved measurement method of the optical reference is feasible and effective, and that the evaluation criterion for the ground validation system is obtained. The evaluation accuracy is improved from 10.99 arc-seconds to 4.88 arc-seconds under the condition of remaining the primary ground test system

V. CONCLUSIONS

The ground test is necessary in order to achieve a more precise attitude determination for the near-earth flight vehicle. In general, the arc-seconds accuracy level ground test is based on the direct astronomical which makes the measurement results more authoritative and authentic. Aiming to propose an efficient ground test method in the lab, this paper investigates two important issues: the improved measurement method for the star simulators intersection angle and the total limit error of the ground validation system.

The equivalent north reference error and intersection angle error are considered as description of the star simulator azimuth error. Using the decoupled attitude adjustment path, simulations are verifying the feasibility and effectiveness for the characteristics of the star simulator azimuth error under the condition of that the gyro errors and the installation error of the star sensor is well estimated. Experimental tests confirm that the error analysis and control method is effective and the verification accuracy coincides with the analyzed evaluation criterion.

REFERENCES

- [1] Wang X, Wang B, Li H. An autonomous navigation scheme based on geomagnetic and starlight for small satellites[J]. *Acta Astronautica*, 2012, 81(1):40-50.
- [2] Zhang H, Zheng W, Tang G. Stellar/inertial integrated guidance for responsive launch vehicles[J]. *Aerospace Science & Technology*, 2012, 18(1):35-41.
- [3] Shen Chong, Song Rui, Li Jie, et al. Temperature drift modeling of MEMS gyroscope based on genetic-Elman neural network[J]. *Mechanical systems and signal processing*, VOL. 72-73, PP. 897-905, 2016
- [4] Zhang Yu, Shen Chong, Tang Jun, Liu Jun. Hybrid Algorithm Based on MDF-CKF and RF for GPS/INS System During GPS Outages[J]. *IEEE Access*, vol. 6, pp. 35343-35354, 2018.
- [5] Sun, T., Xing, F., You, Z. & Wei, M. Motion-blurred star acquisition method of the star sensor under high dynamic conditions[J]. *Opt.Express*. 21, 20096-20110 (2013).
- [6] Jørgensen, J. L., Jørgensen, P. S., Betto, M., Denver, T. & Tuñón, L. J. Enhanced mission performance from autonomous instrument guidance[J]. *Acta Astronaut.* 59, 981-989 (2006).
- [7] S.P. Worden, R.R. Correll, Responsive space and strategic information, *Defense Horizons* 40 (April 2004) 1-8.
- [8] M.J. Veth, J.F. Raquet, Alignment and calibration of optical and inertial sensors using stellar observations[C]. *Proceedings of ION GNSS 2005*, Sep. 2005, pp. 2494-2503.
- [9] Y. Lai, J. Liu, Y. Ding, D. Gu, and D. Yi, Attitude aberration correction for space technology experiment and climate exploration (STECE) satellite star sensor[C]. *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.* 229(6), 1142-1153 (2015).
- [10] T. Martin, U. Probst, H. Fischer, J. Straub-Kalthoff and U. Herberth, Simulation tool chain for multi-sensor navigation filters. 2014 DGON Inertial Sensors and Systems (ISS), Karlsruhe, 2014, pp. 1-16.
- [11] Wei Q, Fang J, Fan X, et al. Hybrid simulation system study of SINS/CNS integrated navigation[J]. *Aerospace & Electronic Systems Magazine IEEE*, 2008, 23(2):17-24.
- [12] Theil S, Steffes S, Samaan M, et al. Hybrid Navigation System for Spaceplanes, Launch and Re-Entry Vehicles[C]. *International Space Planes and Hypersonic Systems and Technologies Conference*. DLR, 2013.
- [13] Brady T, Buckley S, Tillier C. Ground validation of the inertial stellar compass[C]. *Aerospace Conference*, 2004. *Proceedings. IEEE*, 2007:226 Vol.1.
- [14] Rogers R M. *Applied Mathematics in Integrated Navigation Systems*, Third Edition[J]. *Reston American Institute of Aeronautics & Astronautics Inc*, 2007(2):78.
- [15] Wang Q, Diao M, Gao W, et al. Integrated navigation method of a marine strapdown inertial navigation system using a star sensor[J]. *Measurement Science & Technology*, 2015, 26(11):115101. 12S.P. Worden, R.R. Correll, Responsive space and strategic information, *Defense Horizons* 40 (April 2004) 1-8.
- [16] Zhang S, Xing F, Sun T, et al. Novel approach to improve the attitude update rate of a star tracker[J]. *Optics Express*, 2018, 26(5):5164.
- [17] Sun T, Fei X, Wang X, et al. An accuracy measurement method for star sensors based on direct astronomic observation[J]. *Scientific Reports*, 2016, 6:22593.
- [18] Zhang G, Jiang J, Yan J. Dynamic imaging model and parameter optimization for a star tracker[J]. *Optics Express*, 2016, 24(6):5961-5983.
- [19] Zhang G, Jie J, Yan J, et al. Star centroiding error compensation for intensified star sensors[J]. *Optics Express*, 2016, 24(26):29830.
- [20] Zhan D, Hu F, Jiang G, et al. Attitude-correlated frames approach for a star sensor to improve attitude accuracy under highly dynamic conditions[J]. *Applied Optics*, 2015, 54(25):7559-66. New method assessment
- [21] Zheng X, Wei Z, Zhang G. Angular distance constraints calibration for outdoor zoom camera[J]. *Optics Express*, 2016, 24(21):23898.
- [22] Yang B, Su H, Wang M, et al. On-orbit calibration approach for optical navigation camera in deep space exploration[J]. *Journal of Deep Space Exploration*, 2016, 24(5):5536.
- [23] Fan C, Wang M, Yang B, et al. A Method of High-Precision Ground Processing for Star Sensor and Gyro Combination and Accuracy Verification[J]. *Acta Optica Sinica*, 2016.
- [24] Y. Yang, C. Zhang, J. Lu and H. Zhang, Classification of Methods in the SINS/CNS Integration Navigation System[J]. *IEEE Access*, 2018 (6):3149-3158.
- [25] Badshah, K.; Yongyuan, Q.; Zhang, J. SINS/CNS integration algorithm and simulations for extended time flights using linearized Kalman Filtering. In *Proceedings of the 2015 IEEE International Conference on Communication Software and Networks (ICCSN)*, Chengdu, China, 6-7 June 2015; pp. 33-37.
- [26] Yang Y, Zhang C, Lu J, et al. In-flight Calibration of Gyros and Star Sensor with Observability Analysis for SINS/CNS Integration[J]. *IEEE Sensors Journal*, 2017, PP(99):1-1.

Yanqiang Yang was born in 1990. He received a Bachelor's degree from North University of China, Taiyuan, China. Currently, he is studying for his PhD at Beijing University of Aeronautics and Astronautics. His research interests are related to inertial navigation and integrated navigation.

Chunxi Zhang was born in 1965. He received a PhD from Zhejiang University. He has been an professor at Beijing University of Aeronautics and Astronautics. His current research interests are in integrated navigation and optical fiber sensing technology.

Jiazhen Lu was born in 1982. He received a PhD from the School of Instrumentation Science and Opto-electronics Engineering (SIOE) at Beijing University of Aeronautics and Astronautics, Beijing, China, in 2009. Since 2009, he has been an instructor at Beijing University of Aeronautics and

Astronautics. His current research interests are integrated navigation and control engineering.

Hao Zhang was born in 1989. He received a Bachelor's degree from Southeast University, Nanjing, China. Currently, he is studying for his PhD at Beijing University of Aeronautics and Astronautics. His research interests are related to star sensor.